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New solutions to the classical non-Abelian Yang–Mills gauge field with confinement

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Abstract. Within the differential geometry regime, we have introduced an ansatz and have reduced the classical O(4) non-Abelian Yang-Mills gauge field equations into one differential equation which essentially describes the variation of the field components in generalised time and space. This ansatz amounts to requiring that the metric tensor be diagonal and the two parameter functions be functions of radial distance only. We have carried out a series of numerical simulations of the field equation and found that the field strength components display the confinement property.

1. Introduction

Much effort has been expended in the past few decades towards trying to obtain solutions [1, 2] to the classical non-Abelian Yang-Mills gauge field. An early solution, namely a monopole with singular string, was found in 1968 [3]. Monopoles with different magnetic charges were deduced later [4-6]. Within the Euclidean SU(2) gauge regime, the instanton solution was found [7, 8]. Such a solution is non-singular, localised, self-dual, and has a topological charge of one unit. Later, multi-instantons [9, 10] were derived which have square integrable gauge potential. Another type of solution, the meron [11], was then found to represent a point-like concentration of $\frac{1}{2}$ unit of topological charge, with a non-zero rest mass. Then multi-merons [12] were found to be derivable from the YM gauge field. Some other solutions are also known to exist.

When we attempt to solve the gauge field equation, we often look for solutions which might represent particles participating in physical interactions. For that reason, soliton-type solutions are much more desirable. Though monopoles and instantons are viewed as solitons, they do not show propagating characteristics. A method commonly used in obtaining solutions to the complex nonlinear field equation is the introduction of an ansatz.

In this paper, we study the classical solutions of the YM gauge field equation in Euclidean spacetime within the differential geometry regime. These solutions are useful as they correspond to an approximate description of quantum tunnelling effect [1, 2]. Notice, though, that the spacetime manifold is flat when the special solutions are obtained. In looking for a special solution, we shall assign a set of ansatzë to the line element under the spherically symmetric condition. Eventually we arrive at only one gauge field equation describing the variation of an O(4) non-Abelian gauge field. Though we have not been able to obtain analytical solutions to this field equation, we have carried out a series of numerical simulations. We would like to report some very interesting features of these solutions to the field strength components. In particular, the 'confinement property' appears to persist in the solutions.

A similar method is applied to obtain soliton solutions using a different set of ansatzë and the result is published in another paper [13].

2. One type of spherically symmetrical gauge field

The method we use in our derivation is to start with a line element or metrix tensor in four-dimensional space. Employing a set of orthogonal unit basis frame transformations, we obtain a vierbein field [14] leading to a spin connection [15, 16]. We state that such a spin connection is simply our gauge potential. Our ansatz is to take that metrix tensor $g_{\mu\nu}$ be diagonal. A similar method has recently been applied to find new soliton solutions to the non-Abelian YM gauge field [13].

Consider a system with spherical symmetry, so that the line element ds in four-dimensional space can be expressed as

$$\begin{aligned} ds^2 &= -g_{\mu\nu}\, dx^\mu\, dx^\nu \\ &= -e^{2\Phi}\, d\tau^2 - e^{2\Lambda}\, dr^2 - r^2(d\theta^2 + \sin^2\theta\, d\phi^2) \end{aligned} \tag{2.1}$$

where $\tau = it$, $c = 1$, and

$$g_{\mu\nu} = \begin{pmatrix} e^{2\Lambda} & & & 0 \\ & r^2 & & \\ & & r^2\sin^2\theta & \\ 0 & & & e^{2\Phi} \end{pmatrix}$$

with $\mu, \nu = 1, 2, 3, 4$; $x^\mu = (r, \theta, \phi, \tau)$, Λ and Φ being functions of r only, and other symbols have their usual meanings. In order to study the gauge field properties, we introduce an orthonormal frame with unit basis $dy^{\hat{\alpha}}$ which is related to the spacetime variable by the following transformation:

$$dy^{\hat{\alpha}} = L^{\hat{\alpha}}_{\mu}\, dx^\mu \tag{2.2}$$

where $L^{\hat{\alpha}}_{\mu}$ is the vierbein field and $\hat{\alpha} = 1, 2, 3, 4$ is a group index. As pointed out by Wilczek [17], the gauge field properties can be analysed more clearly by introducing the vierbein field. Based on (2.2), the metric tensor becomes

$$g_{\mu\nu} = L^{\hat{\alpha}}_{\mu} L^{\hat{\beta}}_{\nu} \delta_{\hat{\alpha}\hat{\beta}} \tag{2.3}$$

and we have the orthogonal condition

$$L^{\hat{\alpha}}_{\mu} L^{\hat{\beta}\mu} = \delta^{\hat{\alpha}\hat{\beta}}. \tag{2.4}$$

In view of the diagonal form of $g_{\mu\nu}$ in (2.1) under the spherical symmetry condition, the vierbein field is simply

$$L^{\hat{\alpha}}_{\mu} = \begin{pmatrix} e^{\Lambda} & & & 0 \\ & r & & \\ & & r\sin\theta & \\ 0 & & & e^{\Phi} \end{pmatrix} \tag{2.5}$$

which is related to the spin connection $C^{\hat{\alpha}\hat{\beta}}_{\mu}$ by [17]

$$C^{\hat{\alpha}\hat{\beta}}_{\mu} = \tfrac{1}{2} L^{\lambda\hat{\beta}}(L^{\hat{\alpha}}_{\mu,\lambda} - L^{\hat{\alpha}}_{\lambda,\mu}) + \tfrac{1}{2} L^{\hat{\alpha}\sigma}(L^{\hat{\beta}}_{\sigma,\mu} - L^{\hat{\beta}}_{\mu,\sigma}) + \tfrac{1}{2} L^{\hat{\alpha}\sigma} L^{\lambda\hat{\beta}}(L^{\hat{\varepsilon}}_{\sigma,\lambda} L^{\hat{\varepsilon}}_{\mu} - L^{\hat{\varepsilon}}_{\lambda,\sigma} L^{\hat{\varepsilon}}_{\mu}). \tag{2.6}$$

We assert that the spin connection $C_\mu^{\hat\alpha\hat\beta}$ is simply our O(4) gauge potential, i.e. the potential is $A_\mu^{\hat\alpha\hat\beta} = C_\mu^{\hat\alpha\hat\beta}$. In polar coordinates, it is then elementary to show that the non-zero components of the potential appear as

$$\begin{aligned} A_\phi^{\hat r\hat\phi} &= -\mathrm{e}^{-\Lambda}\sin\theta \qquad & A_\theta^{\hat r\hat\theta} &= -\mathrm{e}^{-\Lambda} \\ A_\phi^{\hat\theta\hat\phi} &= -\cos\theta \qquad & A_\tau^{\hat r\hat\tau} &= -\mathrm{e}^{\Phi-\Lambda}\Phi' \end{aligned} \tag{2.7}$$

where the prime indicates differentiation with respect to r. It is well known that the O(4) gauge field strength $F_{\mu\nu}^{\hat\alpha\hat\beta}$ is expressible in terms of $A_\mu^{\hat\alpha\hat\beta}$ and its derivative:

$$F_\mu^{\hat\alpha\hat\beta} = \partial_\mu A_\nu^{\hat\alpha\hat\beta} - \partial_\nu A_\mu^{\hat\alpha\hat\beta} + A_\mu^{\hat\alpha\hat\sigma} A_\nu^{\hat\sigma\hat\beta} - A_\nu^{\hat\alpha\hat\sigma} A_\mu^{\hat\sigma\hat\beta}. \tag{2.8}$$

From (2.7) and (2.8), we obtain the non-zero components of the field strength:

$$\begin{aligned} &F_{r\theta}^{\hat r\hat\theta} = \mathrm{e}^{-\Lambda}\Lambda' \qquad F_r^{\hat r\hat\phi} = \mathrm{e}^{-\Lambda}\Lambda'\sin\theta \qquad F_{r\tau}^{\hat r\hat\tau} = -(\mathrm{e}^{\Phi-\Lambda}\Phi')' \\ &F_{\theta\phi}^{\hat\theta\hat\phi} = \sin\theta(1-\mathrm{e}^{-2\Lambda}) \qquad F_{\theta\tau}^{\hat\theta\hat\tau} = -\mathrm{e}^{\Phi-2\Lambda}\Phi' \qquad F_{\phi\tau}^{\hat\phi\hat\tau} = -\mathrm{e}^{\Phi-2\Lambda}\Phi'\sin\theta \end{aligned} \tag{2.9}$$

$$\begin{aligned} &F^{\hat r\hat\theta\hat r\hat\theta} = g^{rr}g^{\theta\theta}F_{r\theta}^{\hat r\hat\theta} = \frac{\mathrm{e}^{-\Lambda}\Lambda'}{r^2} \qquad F^{\hat r\hat\phi r\phi} = g^{rr}g^{\phi\phi}F_{r\phi}^{\hat r\hat\phi} = \frac{\mathrm{e}^{-\Lambda}\Lambda'}{r^2\sin\theta} \\ &F^{\hat r\hat\tau r\tau} = g^{rr}g^{\tau\tau}F_{r\tau}^{\hat r\hat\tau} = -(\mathrm{e}^{\Phi-\Lambda}\Phi')' \qquad F^{\hat\theta\hat\phi\theta\phi} = g^{\theta\theta}g^{\phi\phi}F_{\theta\phi}^{\hat\theta\hat\phi} = \frac{(1-\mathrm{e}^{-2\Lambda})}{r^4\sin\theta} \\ &F^{\hat\theta\hat\tau\theta\tau} = g^{\theta\theta}g^{\tau\tau}F_{\theta\tau}^{\hat\theta\hat\tau} = \frac{\mathrm{e}^{\Phi-2\Lambda}\Phi'}{r^2} \qquad F^{\hat\phi\hat\tau\phi\tau} = g^{\phi\phi}g^{\tau\tau}F_{\phi\theta}^{\hat\phi\hat\tau} = \frac{\mathrm{e}^{\Phi-2\Lambda}\Phi'}{r^2\sin\theta}. \end{aligned} \tag{2.10}$$

On the other hand, in flat spacetime, the gauge field equation is

$$\partial_\nu F^{\hat\alpha\hat\beta\mu\nu} + A_\nu^{\hat\alpha\hat\lambda}F^{\hat\lambda\hat\beta\mu\nu} + A_\nu^{\hat\beta\hat\lambda}F^{\hat\alpha\hat\lambda\mu\nu} + \Gamma^\mu_{\sigma\nu}F^{\hat\alpha\hat\beta\sigma\nu} + \Gamma^\nu_{\sigma\nu}F^{\hat\alpha\hat\beta\mu\sigma} = 0. \tag{2.11}$$

Note that the last two terms are zero in a cartesian coordinate system; here they are non-zero in the polar coordinate system. It is easy to write down the explicit expressions for the Christoffel symbol $\Gamma^\alpha_{\mu\nu}$ in flat spacetime and we shall omit the procedure here. Using such expressions, putting (2.7) and (2.10) into (2.11), we arrive at a system of gauge field equations in the spherically symmetric case

$$(\mathrm{e}^{\Phi-\Lambda}\Phi')'' + \frac{2}{r}(\mathrm{e}^{\Phi-\Lambda}\Phi')' - \frac{2\mathrm{e}^{-2\Lambda}}{r^2}(\mathrm{e}^{\Phi-\Lambda}\Phi') = 0 \tag{2.12a}$$

$$(\mathrm{e}^{-\Lambda}\Lambda')' - \frac{1}{r^2}\mathrm{e}^{-\Lambda}(1-\mathrm{e}^{-2\Lambda}) + \mathrm{e}^{-\Lambda}(\mathrm{e}^{\Phi-\Lambda}\Phi')^2 = 0. \tag{2.12b}$$

3. Solutions with confined properties

A simple ansatz for the system (2.12*a*) and (2.12*b*) is given by $\Phi = 0$. We can simplify this system to one equation

$$(\mathrm{e}^{-\Lambda}\Lambda')' - \frac{1}{r^2}\mathrm{e}^{-\Lambda}(1-\mathrm{e}^{-2\Lambda}) = 0. \tag{3.1}$$

Defining $f(r) \equiv \mathrm{e}^{-\Lambda(r)}$, (3.1) can be simplified further to

$$f'' + \frac{1}{r^2}f(1-f^2) = 0. \tag{3.2}$$

Note that under the condition $\Phi = 0$, the non-zero components of the gauge potential and field strength become simply

$$A_\theta^{\hat{r}\hat{\theta}} = -f \qquad A_\phi^{\hat{r}\hat{\phi}} = -f \sin\theta \qquad A_\phi^{\hat{\theta}\hat{\phi}} = -\cos\theta \tag{3.3}$$

$$F^{\hat{r}\hat{\theta}r\theta} = \frac{1}{r^2} f' \qquad F^{\hat{r}\hat{\phi}r\phi} = \frac{1}{r^2 \sin\theta} f' \qquad F^{\hat{\theta}\hat{\phi}\theta\phi} = \frac{1}{r^4 \sin\theta}(1 - f^2). \tag{3.4}$$

4. Analysis

In the last section, we have arrived at a field equation of the form specified in (3.2). In order to study the special features of f and hence the potential $A_\mu^{\hat{\alpha}\hat{\beta}}$ in a convenient way, we carry out another transformation

$$r \equiv r_0 \, \mathrm{e}^{-z} \tag{4.1}$$

and write the normalised field equation as

$$\ddot{f} + \dot{f} + f(1 - f^2) = 0 \tag{4.2}$$

with $\dot{f} \equiv \mathrm{d}f/\mathrm{d}z$. Writing (4.2) as $\ddot{f} = -\dot{f} - \partial V/\partial f$, we retrieve the Newtonian equation of motion for a particle experiencing a damping force $-f\dot{f}$ and a conservative force derivable from a potential V; here

$$V = \tfrac{1}{2} f^2 - \tfrac{1}{4} f^4 \tag{4.3}$$

In figure 1, we show the variation of this potential V with respect to f, which is treated as a generalised coordinate. Let us first neglect the influence of the damping force on the motion of the particle. As the 'hypothetical particle' travels from $f = -\infty$, it passes through the peak specified by $f = -1$. Then the particle goes down the slope, passing through the origin $f = 0$ to the other side. In case (i), if the velocity of the particle at P_1 is zero, and it is allowed go to the right then it will be momentarily at rest at P_2. In practice, it will oscillate between P_1 and P_2. If, however, in case (ii), the velocity is greater than 0 at P_1, the particle will pass through P_2 and go down the slope to the right side. As $f \to \infty$, $V \to -\infty$. Thus the initial condition determines the type of motion of the particle. If we include the damping force in case (i), the particle will oscillate

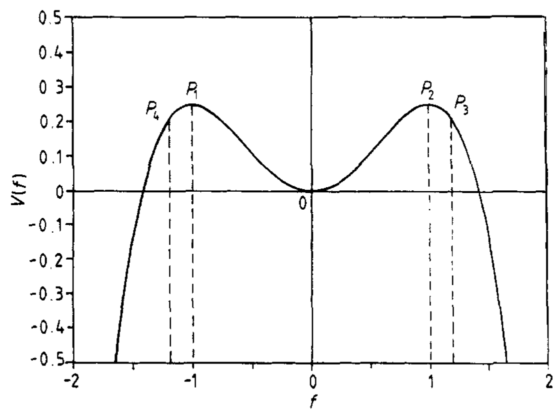

Figure 1. The conservative potential $V(f)$ as a function of the generalised coordinate f.

about 0 with decreasing amplitude. In case (ii), if the velocity at P_1 is large enough, the particle could still go beyond P_2 and down the slope on the positive side of f. If the velocity is not large enough to overcome the influence of damping, the particle could not go beyond P_2, but would again oscillate about 0, but for a longer 'time' interval.

We cannot obtain an analytical solution to equation (4.2). Using numerical simulation, we show in figure 2, the evolution of f and $\dot{f}$ in the course of 'generalised time' z, under the boundary condition that the generalised velocity is $\dot{f}=0$, $f=0.4$ at $z=0$. Referring back to figure 1, the particle passes through point P_1, 0, to reach the point $f=0.4$ where the particle is momentarily at rest, and it comes back and oscillates. We see in figure 2 that $f\to 0$ as z increases. The velocity $\dot{f}$ is also plotted for convenience of analysis. Both f and $\dot{f}$ pass through the horizontal axis an infinite number of times. In fact, the periodicity of oscillation of f can be obtained in the 'small-f domain', so that

$$|f| \gg |f^3|$$

and the field equation is approximately given by

$$\ddot{f}+\dot{f}+f=0 \tag{4.4}$$

giving a solution

$$f=A\,\mathrm{e}^{-z/2}\cos(\tfrac{1}{2}\sqrt{3}z+\phi_0) \tag{4.5}$$

where A and ϕ_0 are integration constants. The period of the oscillation is $4\pi/\sqrt{3}$ in this case.

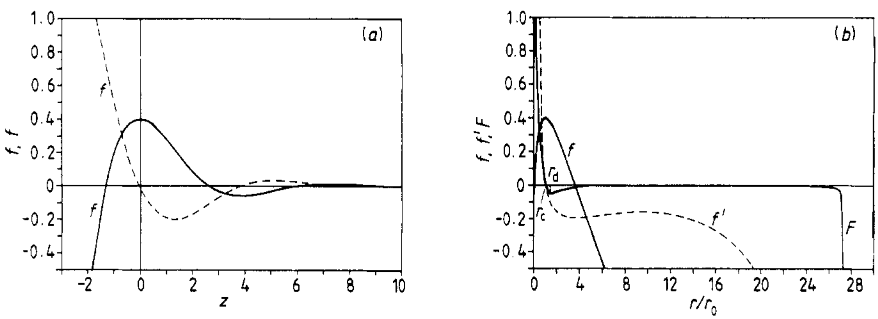

Figure 2. (a) Variation of the functions f and $\dot{f}=\mathrm{d}f/\mathrm{d}z$ with generalised time z, under the boundary conditions $f=0.4$, $\mathrm{d}f/\mathrm{d}z=0$ at $z=0$. (b) Dependence of f, $f'=\mathrm{d}f/\mathrm{d}r$ and F on normalised distance r/r_0 with the same boundary conditions.

In order to visualise how the relevant functions vary with changing radial distance r, we plot in figure 2(b) the graphs for $f(r)$, $f'(r)$ and $F(r)$ which represents the component $F^{\hat{r}\hat{\theta}r\theta}$:

$$F(r)=F^{\hat{r}\hat{\theta}r\theta}=\frac{1}{r^2}f' \tag{4.6}$$

where $f'\equiv \mathrm{d}f/\mathrm{d}r$.

Note that in figure 2(b), the independent variable has been transformed back to r according to (4.1). We have found that the 'depth of the small well' pertaining to the $F(r)$ curve access at $r_\mathrm{d}\simeq 1.47$. It is worth remarking that $F\to -\infty$ at $r_\mathrm{M}<28$. Since

$F(r)$ is one field strength component, it is connected with the interaction force causing the field. If we put a test particle in the field, its motion is confined within a range $r_c \leqslant r < r_m$; the particle can never go beyond r_m. Another interesting feature is that $F(r) \to -\infty$ at some finite space point in each case, representing the fact that the field is enormous, but attractive in nature. So far, $F(r)$ is not yet a direct physically measurable quantity, yet the 'confined feature' remains after $F(r)$ is transformed. The detail of the connection between $F(r)$ and an experimentally measurable quantity is rather complex, and awaits further research.

As another example, we take the boundary conditions to be: at $z=0$, $f=0$, $\dot{f}=\mathrm{d}f/\mathrm{d}z=1.0$. In figure 3($a$), we plot f and $\dot{f}$ against z, similar to figure 2(a). In figure 3(b), we plot f, f' $(=\mathrm{d}f/\mathrm{d}r)$ and $F(r)$ against r. We note that the 'well' in the $F(r)$ curve is much more pronounced, due to different scaling in r (see (4.1)).

In figure 4, we start with a different type of boundary conditions: at $z=0$, $f=0$, $\mathrm{d}f/\mathrm{d}z=1.5$, such a condition correspond to a different type of motion all together. Refer back to figure 1, the hypothetical particle at point 0 has a rather large velocity, so that it can pass through point P_2 and go down the slope as $V(f) \to -\infty$. In the f scale, the particle starts from $f \to -\infty$, and propagates in the direction shown by the arrow in figure 4. When $f=0$, $r/r_0=1$ and as $r/r_0 \to 0$, $f \to \infty$. We also show the f' curve in figure 4. The behaviour of $F(r)$ is rather special in this case; it remains negative for all spatial points. The particle is being attracted at all points; in particular, it experiences an infinite field strength as r/r_0 falls below a critical value $\sim$4.3.

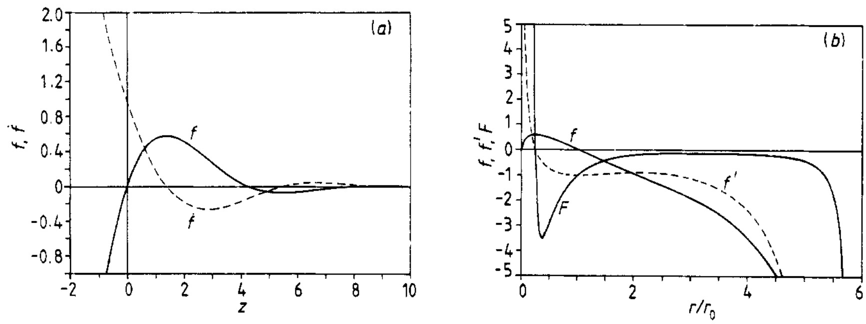

Figure 3. (a) Variation of the functions f, $\dot{f}=\mathrm{d}f/\mathrm{d}z$ with generalised time z, under the boundary conditions $f=0$, $\mathrm{d}f/\mathrm{d}z=1$ at $z=0$. (b) Dependence of f, $f'=\mathrm{d}f/\mathrm{d}r$ and F on normalised distance r/r_0 with the same boundary conditions.

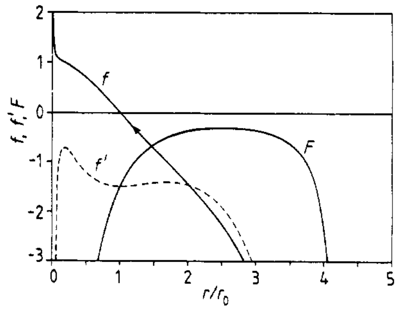

Figure 4. f, $f'=\mathrm{d}f/\mathrm{d}r$ and F plotted against r/r_0 under the boundary conditions $f=0$, $\mathrm{d}f/\mathrm{d}z=1.5$ at $z=0$.

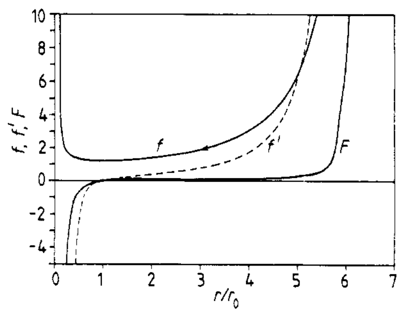

Figure 5. $f, f'=\mathrm{d}f/\mathrm{d}z$ and as functions of r/r_0 under the boundary conditions $f=1.2$, $\mathrm{d}f/\mathrm{d}z=0$ at $z=0$.

The third class of solution is displayed in figure 5 with the following boundary conditions: at $z=0, f=1.2, \mathrm{d}f/\mathrm{d}z=0$. Refer back to figure 1: the particle is travelling towards the left and is at rest at P_3 ($f=1.2$). It then moves 'backwards' in the sense that it is being repelled by the field and goes to $f\to\infty$. The particle can never reach the domain bounded by P_1 and P_2.

Following the same type of argument, if the particle starts at P_4 ($f=-1.2, \mathrm{d}f/\mathrm{d}z=0$), it will 'go back' and can never again reach the domain bounded by P_1 and P_2. If we carry out the transform $f\to -f$, $f'\to -f'$, $F=-F$ in figure 5, we will obtain another corresponding figure; we would omit such an extension.

5. Conclusions

(i) Using a method recently developed [14] within the differential geometry regime, we arrive at a single differential equation describing an O(4) non-Abelian YM gauge field using a certain spherically symmetric ansatz.

(ii) We have carried out a series of numerical simulations of the spatial field equation $f''+(1/r^2)f(1-f^2)=0$, where $f'\equiv \mathrm{d}f/\mathrm{d}r$, and the corresponding generalised temporal field equation $\ddot{f}+\dot{f}+f(1-f^2)=0$, where $\dot{f}\equiv \mathrm{d}f/\mathrm{d}z$, $r/r_0\equiv \mathrm{e}^{-z}$. Though the field quantities have yet to be transformed to physically observed quantities, we know that the motion of a particle in a field is related to the field strength tensor as well as the general charge. Drawing an analogue with Newtonian motion, we can imagine that an hypotheical particle in the gauge field is experiencing a conservative potential $V(f)$ as depicted in figure 1 plus a damping force. Based on such an analysis, we have analysed numerically the generalised temporal evolution of $f, f', \dot{f}$ given certain sets of boundary conditions. The distributions of these three functions, together with the field strength $F^{\hat{r}\hat{\theta}r\theta}$ in space r are also analysed numerically.

(iii) Figures 2-5 describe three types of motion of the hypothetical particle: (a) the particle is oscillating between P_1P_2 in figure 1, (b) the particle goes through $P_1, P_2, P_3, \ldots$, (c) the particle proceeds from right to left towards P_3 and then retreats back.

(iv) We observe that under certain boundary conditions the particle experiences a negative infinite field strength at a certain space point. On the other hand, the same field strength is enormously large but positive at a certain smaller value of the space point. Such a feature implies spatial confinement.

(v) If the generalised velocity is large enough (figure 4), $F(r)\to -\infty$ at two space points. Such a result indicates another type of confinement.

(vi) We have numerically analysed the other field strength components and have obtained similar confinement properties. We shall not present similar graphs in in order to save space.

(vii) Our result appears to indicate that further analysis of the field strength and gauge potential components is desirable, in order to apply the theory to specific problems.

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